추정식 도출

$$\log \tilde{y}(t)_i = \left(1-e^{-\gamma t}\right)\log \tilde{y}_i^* + e^{-\gamma t}\log \tilde{y}(0)_i \text{, } i=1,\dots,T$$

$$\Rightarrow \begin{aligned} &\log y(t)_i - \log A(t)_i \\ &= \left(1-e^{-\gamma t}\right)\log y^*{}_i - \left(1-e^{-\gamma t}\right)\log A^*{}_i + e^{-\gamma t}\log y(0)_i - e^{-\gamma t}\log A(0)_i \end{aligned}$$

$$\Rightarrow \log y(t)_i = \left(1-e^{-\gamma t}\right)\log y^*{}_i + e^{-\gamma t}\log y(0)_i + a' + \varepsilon_i'$$

(⇐ 특정시점에서 t는 고정된 상수, 생산성은 모두 확률변수로 간주)

$$\Rightarrow \log y(t)_i - \log y(0)_i = \left(1-e^{-\gamma t}\right)\log y^*{}_i - \left(1-e^{-\gamma t}\right)\log y(0)_i + a' + \varepsilon_i'$$

$$\Rightarrow \begin{aligned} &\log y(t)_i - \log y(0)_i \\ &= \left(1-e^{-\gamma t}\right)\left(a + \frac{\alpha}{1-\alpha-\beta}\ln s_{ki} + \frac{\beta}{1-\alpha-\beta}\ln s_{hi} - \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(n_i+g+d\right) + \varepsilon_i\right) \\ &\quad - \left(1-e^{-\gamma t}\right)\log y(0)_i + a' + \varepsilon_i' \end{aligned}$$

(⇐ 앞서 국가간 소득 차이에 대한 회귀식 ($\log y^*{}_i = a + \frac{\alpha}{1-\alpha-\beta}\ln s_{ki} + \frac{\beta}{1-\alpha-\beta}\ln s_{hi} - \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(n_i+g+d\right) + \varepsilon_i$)을 이용함)

$$\begin{aligned} &= \left(1-e^{-\gamma t}\right)\left(\frac{\alpha}{1-\alpha-\beta}\ln s_{ki} + \frac{\beta}{1-\alpha-\beta}\ln s_{hi} - \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(n_i+g+d\right)\right) \\ &\quad - \left(1-e^{-\gamma t}\right)\log y(0)_i + \beta_0 + \mu_i \end{aligned}$$

(⇐ $\beta_0 = \left(1-e^{-\gamma t}\right)a + a'$, $\mu_i = \left(1-e^{-\gamma t}\right)\varepsilon_i + \varepsilon_i'$ ←특정시점에서 t는 고정된 상수)

$$\Rightarrow \begin{aligned} &\log y(t)_i - \log y(0)_i \\ &= \beta_0 + \beta_1 \ln s_{ki} + \beta_2 \ln s_{hi} + \beta_3 \ln\left(n_i+g+d\right) + \beta_4 \log y(0)_i + \mu_i \end{aligned}$$