A. $Y = K^{\alpha} H^{\beta} \left(AL \right)^{1-\alpha-\beta}$, $\tilde{y} = \tilde{k}^{\alpha} \tilde{h}^{\beta}$

이윤극대화의 일계조건 : $r_K = \alpha \dfrac{Y}{K} = \alpha \dfrac{\tilde{y}}{\tilde{k}}$, $r_H = \beta \dfrac{Y}{H} = \beta \dfrac{\tilde{y}}{\tilde{h}}$

Arbitrage(차익거래)가 없음 : $r_K = r_H \Rightarrow \tilde{h} = \dfrac{\beta}{\alpha} \tilde{k} \;\Rightarrow\; \tilde{y} = \lambda \tilde{k}^{\alpha+\beta}$, $\lambda = \left(\dfrac{\beta}{\alpha} \right)^{\beta}$

자본축적 방정식 : $\dfrac{\dot{\tilde{k}}}{\tilde{k}} = s_k \lambda \tilde{k}^{\alpha+\beta-1} - \left(n + g + d \right)$

B. 균제상태 근처에서 근사적으로 다음의 식이 성립함(테일러 전개)

($f(x) \cong f(x^*) + f\,'(x)\big|_{x=x^*} (x - x^*)$, $\dfrac{d \log \tilde{k}}{dt} \equiv f(\log \tilde{k}) \equiv f(x) \;\Rightarrow$)

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{d \log \tilde{k}}{dt} \cong \left. \frac{d \log \tilde{k}}{dt} \right|_{\log \tilde{k} = \log \tilde{k}^*} + \left. \frac{d\left(\dfrac{d \log \tilde{k}}{dt} \right)}{d \log \tilde{k}} \right|_{\log \tilde{k} = \log \tilde{k}^*} \left(\log \tilde{k} - \log \tilde{k}^* \right)$$

($\left. \dfrac{d \log \tilde{k}}{dt} \right|_{\log \tilde{k} = \log \tilde{k}^*} = 0 \;\Rightarrow$)

$$= \left. \frac{d}{d \log \tilde{k}} \left[s_k \lambda \tilde{k}^{\alpha+\beta-1} - \left(n + g + d \right) \right] \right|_{\log \tilde{k} = \log \tilde{k}^*} \left(\log \tilde{k} - \log \tilde{k}^* \right)$$

($x = \exp(\log x) \;\Rightarrow$)

$$= \left. \frac{d}{d \log \tilde{k}} \left[s_k \lambda \exp\left(\log \tilde{k}^{\alpha+\beta-1} \right) - \left(n + g + d \right) \right] \right|_{\log \tilde{k} = \log \tilde{k}^*} \left(\log \tilde{k} - \log \tilde{k}^* \right)$$

($\dfrac{d \exp(f(x))}{dx} = f\,'(x) \exp(f(x)) \;\Rightarrow$)

$$= \left(\alpha + \beta - 1 \right) s_k \lambda \tilde{k}^{\alpha+\beta-1} \Big|_{\log \tilde{k} = \log \tilde{k}^*} \left(\log \tilde{k} - \log \tilde{k}^* \right)$$

($s_k \lambda \tilde{k}^{\alpha+\beta-1} \Big|_{\log \tilde{k} = \log \tilde{k}^*} = n + g + d$ (균제상태이므로) $\Rightarrow$)

$$= \left(1 - \alpha - \beta \right) \left(n + g + d \right) \left(\log \tilde{k}^* - \log \tilde{k} \right) \equiv \gamma \left(\log \tilde{k}^* - \log \tilde{k} \right)$$

C. $\tilde{y} = \lambda \tilde{k}^{\alpha+\beta} \Rightarrow \frac{\dot{\tilde{y}}}{\tilde{y}} = (\alpha+\beta)\frac{\dot{\tilde{k}}}{\tilde{k}}$,

$$\log \tilde{y}^* = \log\lambda + (\alpha+\beta)\log\tilde{k}^*,\ \ \log\tilde{y} = \log\lambda + (\alpha+\beta)\log\tilde{k}$$

$$\Rightarrow\ \log\frac{\tilde{y}^*}{\tilde{y}} = (\alpha+\beta)\log\frac{\tilde{k}^*}{\tilde{k}}$$

$$\Rightarrow\ \frac{\dot{\tilde{y}}}{\tilde{y}} = (\alpha+\beta)\frac{\dot{\tilde{k}}}{\tilde{k}} = (\alpha+\beta)\gamma\left(\log\tilde{k}^* - \log\tilde{k}\right)$$

$$= \gamma\left(\log\tilde{y}^* - \log\tilde{y}\right),\ \ \gamma = (1-\alpha-\beta)(n+g+d)$$

$$\Rightarrow\ \frac{d\log\tilde{y}}{dt} = \gamma\left(\log\tilde{y}^* - \log\tilde{y}\right)$$ (1계선형미분방적식)

$$\Rightarrow\ \begin{aligned} \log\tilde{y}(t) &= \log\tilde{y}^* + \left(\log\tilde{y}(0) - \log\tilde{y}^*\right)e^{-\gamma t} \\ &= \left(1-e^{-\gamma t}\right)\log\tilde{y}^* + e^{-\gamma t}\log\tilde{y}(0) \end{aligned}$$